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Einstein Brane-Worlds In 5D Gauged Supergravity

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Abstract

We study, in the context of five dimensional $N = 2$ gauged supergravity with vector and hypermultiplets, curved domain wall solutions with worldvolumes given by four dimensional Einstein manifolds. For a choice of the projection condition on the Killing spinors of the BPS solutions, first order differential equations governing the flow of the scalars are derived. With these equations, we analyze the equations of motion and determine conditions under which gauged supergravity theories may admit Einstein domain wall solutions. These conditions, however, can be reconciled with the integrability conditions of the Killing spinors equations only for vanishing cosmological constant.

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1 Introduction

Recently there has been some interest in the study of supersymmetric as well as non-supersymmetric domain walls and black holes as solutions of $N = 2$ five-dimensional gauged supergravity. This activity has been motivated mainly by the desire to embed the Randall-Sundrum [?] scenario in the framework of string or M-theory. The choice for five dimensional gauged theories comes from the fact that such theories allow for anti-de Sitter vacuum states which are fundamental for the realization of the Randall-Sundrum model. Also of interest is the study of black hole solutions in gauged supergravity theories as they play a fundamental role in the conjectured AdS/CFT correspondence [?]. In the past few years, supersymmetric black holes and strings, as well as non-supersymmetric generalizations, have been constructed (see [?]) for the $U(1)$ -gauged $N = 2$ supergravity [?].

Very recently, there has been a shift towards the study of $N = 2$ supergravity with gauged hypermultiplets. Ultimately, one would like to investigate the most general theories and study their solutions in the hope of finding a particular model which may incorporate Randall-Sundrum scenario in a supersymmetric setting. Flat domain walls and black hole solutions for five dimensional supergravity theories with gauged isometries of the hypermultiplets have been discussed very recently in [?, ?, ?].

In this paper we are interested in the study of curved Einstein domain walls for the gauged supergravity theories discussed in [?]. Such a study has been initiated by the recent work of Cardoso et al [?]. In a previous paper [?], we have shown that the flat-worldvolume domain wall solutions found in [?] can be generalized to solutions with Ricci-flat worldvolumes. In the work of [?] it was established that flat BPS domain wall solutions of gauged supergravity with hypermultiplets can only exist under certain conditions (see next section). These conditions are in general not satisfied and this may restrict the class of gauged supergravities that have flat BPS solutions. Our purpose in this work is the study of supersymmetric domain wall solutions with four dimensional worldvolumes given by Einstein spaces with a negative cosmological constant. In the analysis of [?], where the projection condition on the Killing spinors as given in [?] was used, it was shown that such solutions do not exist. Later, and in the revised work of [?] a more general projection condition was proposed. We shall show here that the conclusions of [?] remain valid and that supersymmetric Einstein domain wall solutions in presence of non-trivial matter, within the framework of [?], are not allowed. However, non-supersymmetric solutions may be possible if one satisfies the equations of motion and ignores the integrability conditions coming from the Killing spinors equations. The conditions under which the non-supersymmetric solutions of [?] (generalized to many vector and hypermultiplets) can be obtained as solutions of five dimensional supergravity theory are determined. Our analysis is carried out in the context of gauged $N = 2$ supergravity models of [?] and in the absence of tensormultiplets.

We organize this work as follows. In the next section the supergravity theories we wish to study are reviewed together with a brief discussion on their possible Ricci-flat domain wall solutions. In section three, we look for BPS Einstein domain wall solutions

and derive first order differential equations by solving for the vanishing of supersymmetry transformations of the fermi fields in a bosonic background. This is done for a general choice of the projection condition on the Killing spinors which was given in [?]. The equations of motion are analyzed and we determine some conditions under which the models presented in [?] can be embedded in gauged $N = 2$ supergravity theory. We demonstrate that the constraints derived from the equations of motion can not be combined with the integrability conditions (coming from the requirement of unbroken supersymmetry) and thus the solutions do not in general admit supersymmetry. Finally we summarize our results and discuss possible future directions.

2 Ricci-Flat Domain Walls

In this section we briefly review gauged supergravity models and their possible Ricci-flat domain wall solutions [?, ?, ?]. The gauged supergravity theories we are interested in are those minimal theories (with eight real supercharges) coupled to n_V vectormultiplets and n_H hypermultiplets, where global isometries including R -symmetry are made local. Specifically, we consider the models constructed in [?] without tensormultiplets. The fermionic fields of the $N = 2$ supergravity theory are the gravitini ψ_M^i which are symplectic Majorana spinors ($i = 1, 2$ are $SU_R(2)$ indices), the gaugini $\lambda_i^{\hat{a}}$ ² and the hyperini ζ^α ($\alpha = 1, \dots, 2n_H$). The bosonic fields consist of the graviton, vector bosons A_M^I ($I = 0, 1, \dots, n_V$), the real scalar fields ϕ^x , ($x = 1, \dots, n_V$) of the vectormultiplets and the scalars q^X ($X = 1, \dots, 4n_H$) of the hypermultiplets. The scalar fields of the theory live on a manifold $\mathcal{M} = \mathcal{M}_V \otimes \mathcal{M}_H$, which is the direct product of a very special [?] and a quaternionic Kähler manifold [?] with metrics denoted respectively by $g_{xy}(\phi)$ and $g_{XY}(q)$. The target manifold of the scalar fields of the vectormultiplets \mathcal{M}_V is a very special manifold described by an n_V -dimensional cubic hypersurface

$$C_{IJK}h^I(\phi^x)h^J(\phi^x)h^K(\phi^x) = 1 \quad (2.1)$$

of an ambient space parametrized by $n_V + 1$ coordinates $h^I = h^I(\phi^x)$, where C_{IJK} is a completely symmetric constant tensor which defines Chern–Simons couplings of the vector fields. A classification of the allowed homogeneous manifolds can be found in [?]. The quaternionic Kähler manifold can be described in terms of the $4n_H$ -beins $f_{i\alpha}^X$ obeying the relation $g_{XY}f_{i\alpha}^X f_{j\beta}^Y = \epsilon_{ij} C_{\alpha\beta}$, where ϵ_{ij} and $C_{\alpha\beta}$ are the $SU(2)$ and $USp(2n_H)$ invariant tensors respectively.

We are mainly interested in finding bosonic configurations and we display only the bosonic action of the gauged theory as well as the supersymmetry variations of the fermi

¹In this paper, the indices A, B represent five-dimensional flat indices, $A = (a, 5)$. Curved indices are represented by $M = (\mu, z)$.

²Here \hat{a} is the flat index of the tangent space group $SO(n_V)$ of the scalar manifold \mathcal{M}_V .

fields in a bosonic background. The bosonic action for vanishing gauge fields is given by

$$E^{-1}\mathcal{L} = 12R - 12g_{XY}\partial_M q^X \partial^M q^Y - 12g_{xy}\partial_M \phi^x \partial^M \phi^y - \mathcal{V}(\phi, q), \quad (2.2)$$

where $E = \sqrt{-\det g_{MN}}$ and the scalar potential is given by [?]

$$\mathcal{V} = -g^2 [2P_{ij}P^{ij} - P_{ij}^{\hat{a}}P^{\hat{a}ij}] + 2g^2\mathcal{N}_{i\alpha}\mathcal{N}^{i\alpha}.$$

and

$$P_{ij} \equiv h^I P_{Iij}, \quad P_{ij}^{\hat{a}} \equiv h^{\hat{a}I} P_{Iij}, \quad \mathcal{N}^{i\alpha} \equiv \sqrt{64}h^I K_I^X f_X^{\alpha i}. \quad (2.3)$$

Here K_I^X , P_I are the Killing vectors and prepotentials respectively. For details of the gauging and the meaning of the various quantities, we refer the reader to [?].

In a bosonic background, the supersymmetry transformations of the fermi fields in the gauged theory (after dropping the gauge fields contribution) are given by

$$\begin{aligned} \delta\psi_{Mi} &= \mathcal{D}_M \varepsilon_i + i\sqrt{6}g \Gamma_M \varepsilon^j P_{ij}, \\ \delta\lambda_i^{\hat{a}} &= -i2f_x^{\hat{a}} \Gamma^M \varepsilon_i \partial_M \phi^x + g\varepsilon^j P_{ij}^{\hat{a}}, \\ \delta\zeta^\alpha &= -i2f_{iX}^\alpha \Gamma^M \varepsilon^i \partial_M q^X + g\varepsilon^i \mathcal{N}_i^\alpha. \end{aligned}$$

Flat domain walls for the general gauged $N = 2$ supergravity theory without tensor multiplets were considered in [?]. There, it was found that if one writes

$$P^{(r)}(\phi, q) = h^I(\phi) P_I^{(r)}(q) = \sqrt{32}W Q^{(r)}, \quad Q^{(r)} Q^{(r)} = 1, \quad (2.4)$$

where W is the norm (the superpotential) and $Q^{(r)}$ are $SU(2)$ phases of $P^{(r)}(\phi, q)$, then the existence of BPS flat domain wall solutions with metric

$$ds^2 = e^{2U(z)} \eta_{\mu\nu} dx^\mu dx^\nu + dz^2, \quad (2.5)$$

and Killing spinors satisfying

$$\Gamma_z \varepsilon_i = \gamma_5 \varepsilon_i = Q^{(r)} \sigma_{ij}^{(r)} \varepsilon^j, \quad (2.6)$$

will require that $Q^{(r)}$ satisfy the condition

$$\partial_x Q^{(r)} = 0. \quad (2.7)$$

The condition (2.7) then implies that the scalar potential takes a form which guarantees stability [?],

$$\mathcal{V} = g^2(-6W^2 + 92g^{\Lambda\Sigma}\partial_\Lambda W\partial_\Sigma W) \quad (2.8)$$

where Λ, Σ run over all the scalars of the theory. The condition (2.7) is satisfied when there are no hyper scalars but only Abelian vectormultiplets, in which case the $Q^{(r)}$ are constants. Also (2.7) is obviously satisfied when there are no physical vectormultiplets. In general, the condition (2.7) is not satisfied for a generic point on the scalar manifold, and this may restrict the class of gauged theories that have Ricci-flat BPS solutions.

The scalar fields and the warp factor for the flat BPS domain wall solutions are given by [?]

$$\begin{aligned} \phi'^\Lambda &= -3gg^{\Lambda\Sigma}\partial_\Sigma W, & \phi^\Lambda &= (\phi^x, q^X), \\ U' &= gW. \end{aligned}$$

The prime symbol denotes differentiation with respect to the fifth coordinate z . As discussed in [?], the flat BPS domain walls of [?] can be promoted to solutions with Ricci-flat worldvolumes. The amount of supersymmetry preserved by the five dimensional domain wall depends on the amount of supersymmetry preserved by its four dimensional Ricci-flat worldvolume.

3 Einstein Domain Walls

In this section, we are interested in studying domain walls with Einstein worldvolumes with a negative cosmological constant. In [?], it was shown that supersymmetric solutions are not necessarily solutions of the equations of motion (see also [?, ?]). Therefore it is important, in our subsequent analysis, to make sure that possible supersymmetric configurations satisfy the equations of motion. Our main interest is to determine the conditions under which the $N = 2$ gauged supergravity theories may allow for Einstein domain wall solutions. We start our analysis by allowing for a general projection condition on the Killing spinors; therefore, we write [?]

$$\begin{aligned} \gamma_5 \varepsilon_i &= (\mathcal{A}Q^{(r)} + \mathcal{B}N^{(r)}) \sigma_{ij}^{(r)} \varepsilon^j, \\ Q^{(r)}Q^{(r)} &= N^{(r)}N^{(r)} = 1, & \mathcal{A}^2 + \mathcal{B}^2 &= 1, & Q^{(r)}N^{(r)} &= 0, \end{aligned}$$

where all quantities appearing in the projection condition are in general field dependent. If $\mathcal{A} = 1$, one has the projection condition of [?]. For these cases, as mentioned in the previous section, it was found that one must satisfy $\partial_x Q^{(r)} = 0$ in order to obtain Ricci-flat BPS domain wall solutions.

The metric of our curved domain wall can be put in the form

$$ds^2 = e^{2U(z)} g_{\mu\nu}(x) dx^\mu dx^\nu + dz^2, \quad (3.1)$$

and all the dynamical scalar fields of the theory are assumed to depend only on the fifth coordinate z . The non-vanishing spin connections for our metric are given by

$$\begin{aligned}\Omega_{\mu ab}(x, z) &= \omega_{\mu ab}(x), \\ \Omega_{\mu a 5}(x, z) &= U' e^U e_{a\mu}(x).\end{aligned}$$

From the vanishing of the μ -component of the gravitini supersymmetry transformation we obtain

$$\delta\psi_{\mu i} = D_\mu \varepsilon_i + 12e^U \gamma_\mu \left((\mathcal{A}U' - gW)Q^{(r)} + \mathcal{B}U' N^{(r)} \right) \sigma_{ij}^{(r)} \varepsilon^j, \quad (3.2)$$

where we use the projection condition (??) as well as

$$\Gamma_\mu = e^U \gamma_\mu, \quad \Gamma_z = \gamma_5, \quad D_\mu = \partial_\mu + 14\omega_\mu^{ab} \gamma_{ab}. \quad (3.3)$$

The vanishing of the gaugini supersymmetry transformation results in the following equations representing the supersymmetric flow of the vectormultiplets scalars [?]

$$\begin{aligned}\mathcal{A}\phi'^x &= -3gg^{xy}\partial_y W, \\ \mathcal{B}N^{(r)}\phi'^x &= -3gg^{xy}W\partial_y Q^{(r)}.\end{aligned}$$

Notice that these equations generalize the first order differential equation given in [?] to the cases of many vectormultiplets.

From the vanishing of the hyperini supersymmetry transformation, we obtain the supersymmetric flow equation of the hyper scalars. This is given by

$$\left(\mathcal{A}g_{XY} + 2\mathcal{B}\epsilon^{(r)(s)(t)} Q^{(r)} N^{(s)} R_{XY}^{(t)} \right) q'^Y = \mathcal{A}G_{XY} q'^Y = -3g\partial_X W. \quad (3.5)$$

If one requires that the worldvolume Ricci-tensor satisfy

$$R_{\mu\nu}^{(4)} = -12c^2 g_{\mu\nu}, \quad (3.6)$$

then integrability coming from the vanishing of (3.2) will imply [?]

$$(\mathcal{A}U' - gW)^2 + \mathcal{B}^2 U'^2 = 4c^2 e^{-2U}. \quad (3.7)$$

We now turn to the analysis of the equations of motion. The Einstein equations of motion give for the worldvolume Ricci-tensor

$$R_{\mu\nu}^{(4)} = g_{\mu\nu} e^{2U} (4U'^2 + U'' + 23\mathcal{V}). \quad (3.8)$$